Pareto-Optimal Search over Configuration Space Beliefs for Anytime Motion Planning

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Motion Planning on Geometric Roadmaps

[Planning Algorithms, Lavalle 2006]

[Wikipedia]
Challenges for High-DOF robots

Shortest path - execution speedup vs. extra planning time

No explicit representation of obstacles in 3D world

Expensive collision checking for articulated, complex robots
Obtain successively shorter feasible paths (anytime) while minimizing collision checks
Anytime Motion Planning

Finds initial solution and iteratively improves with time

Video of RRT* algorithm
Related Work

Lazy PRM [Bohlin & Kavraki 2000]
Roadmaps with lazy evaluation; search for paths optimistically

Fuzzy PRM [Nielsen & Kavraki 2000]
Assign weights to edges based only on feasibility likelihood

Realtime Informed Path Sampling [Knepper & Mason 2011]
Probabilistically model obstacle locations from collision tests to guide path sampling

Instance-based Learning [Pan et al. 2012]
Use prior collision check results in a $k$-NN model to reason about unknown configurations
Idea 1: C-Space Belief Model

Maps configuration to probability of being free (we call this collision measure)

Inexpensive (compared to perfect collision checker) but uncertain

Updated online with collision check results
Idea 2: Bi-criteria Optimization

Focus on paths which are both (a) short and (b) likely collision-free
Bi-criteria Optimization

Focus on paths which are both (a) short and (b) likely collision-free

Adjust the tradeoff to organize the search for paths

<table>
<thead>
<tr>
<th>Path Length</th>
<th>Collision Measure</th>
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</thead>
<tbody>
<tr>
<td>Less likely in collision but longer</td>
<td></td>
</tr>
<tr>
<td>More likely in collision but shorter</td>
<td></td>
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</tbody>
</table>
Criteria Functions for Edges (Paths)

Length, based on some metric

\[ w_l : E \to [0, \infty) \quad L(\pi) = \sum_{e \in \pi} w_l(e) \]

Collision measure

\[ w_m : E \to [0, \infty) \quad M(\pi) = \sum_{e \in \pi} w_m(e) \]

\[ w_m(e) = -\log(\rho(e)) \]

where \( \rho(e) \) is the probability of edge being free
Pareto Optimality

Pareto frontier - Set of all Pareto optimal points (not worse off in both criteria than any other)
Pareto Optimality

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Expensive to compute explicitly - $O(n!)$
Convex Hull of Frontier

Path Length vs Collision Measure

- $\alpha_1$
- $\alpha_2$
- $\alpha_3$
Minimize convex combination of $L$ and $M$ for candidate paths

$$\arg \min_{\pi} J^\alpha(\pi) = \alpha L(\pi) + (1 - \alpha) M(\pi), \alpha \in [0, 1]$$

Additive over edges - dynamic programming for search

Vary $\alpha$ monotonically from 0 to 1 to organize search

Equivalent to minimizing expected path length under a penalty model
Algorithm - POMP

Input: $G = (V, E), w_l, w_m, x_{\text{start}}, x_{\text{goal}}$

1. Search for path $\pi$ that minimizes $J^\alpha (\alpha = 0 \text{ initially})$
Algorithm - POMP

Input: $G = (V, E), w_l, w_m, x_{\text{start}}, x_{\text{goal}}$

2. If $\pi \equiv \hat{\pi}$ (current best path), increase $\alpha$ and repeat 1
Algorithm - POMP

Input: $G = (V, E), w_l, w_m, x_{\text{start}}, x_{\text{goal}}$

3. Evaluate $\pi$ for feasibility and update model
Algorithm - POMP

Input: $G = (V, E), w_l, w_m, x_{\text{start}}, x_{\text{goal}}$

4. If $\pi$ is feasible, set $\hat{\pi}$ to $\pi$, report solution and increase $\alpha$.
Algorithm - POMP

Input: \( G = (V, E), w_l, w_m, x_{\text{start}}, x_{\text{goal}} \)

5. Repeat 1

Terminate when \( \alpha = 1 \) and no new candidate paths

Optimal Path Found! Algorithm Terminates
Experimental Details

In simulation on HERB\textsuperscript{1} - planning for the right arm (a 7-DOF Barrett WAM manipulator)

6 problems on 50 roadmaps

Collision Model - kNN lookup; weighted average of nearest known configs

First feasible path

Planning time and collision checks required

POMP (standard) against RRT-Connect (OMPL\(^1\)), LazyPRM and POMP (w/out model updates - using only prior)

First feasible path - time

Planning Time (s)

LazyPRM
RRTConnect
Without Model
With Model

P1
P2
P3
Anytime Performance

Path length against cumulative planning time

Against BIT*¹ (OMPL)

POMP

Anytime motion planning algorithm

Bi-criterion search over length and collision measure

Belief model for configuration space collision probabilities
Future Work

Incremental sampling when roadmap fails

More complex belief models - manifolds, topology

Model reuse for multi-query planning that is robust to small environmental changes
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