1 Configuration Space (contd...)

As discussed in the previous class, the configuration space of a manipulator arm with a single revolute joint is a continuum from 0 to 2π (shown in Figure 1). By identifying 0 and 2π, this C-space becomes a circle (achieved in practice by taking the modulus of the angle by 2π). This allows us to remain inside the C-space when, for example, freely spinning the arm in one direction. Each configuration of this robot is represented as a point on the circumference of the circle.

Similarly, for a manipulator with two revolute joints, the configuration space begins as a plane (the variables of the plane are θ₁ and θ₂, the two joint angles of the robot). Again, we identify the angles 0 and 2π for θ₁, causing two parallel edges of the plane to wrap around and connect, forming a cylinder. Once we do the same identification of 0 and 2π for θ₂, this cylinder becomes a torus (shown in Figure 2). In this case θ₁ is represented along the major axis on the torus and θ₂ is represented on each cross-section of the torus. Thus the overall configuration of the robot would be represented as a point on the surface of the torus.

Therefore, C-Space can be represented in a higher dimensional vector space but the basis of the representation is always less than or equal to the DOF’s of the robot. Figures 3, 4 and 5 show the 2D representation of some common 3D topological features. The arrow on each pair of line segment represent the orientation of the alignment that they have in 3D.

Figure 1: shows a manipulator with a single degree of freedom and its corresponding C-Space which is a straight line [0, 2π] wrapped around itself.
Figure 2: Shows a manipulator with two degrees of freedom and its corresponding C-Space which is an XY grid wrapped around itself twice to form a torus.

Figure 3: Shows a planar manipulator who’s C-space is transformed into a cylinder by one identification.

Figure 4: Shows a planar manipulator who’s C-space is transformed into a mobius strip by one identification.
2 Piano Movers’ Problem

The Piano Movers’ Problem is defined as follows (shown in Figure 6):

1. Given a world \( W = \mathbb{R}^2 \) or \( \mathbb{R}^3 \)
2. Obstacle region \( O \subset W \)
3. Robot A and its C-Space \( C \)
4. \( C_{free} = C \setminus C_{obs} \)
5. \( q_I \in C_{free} : \) Initial configuration
6. \( q_G \in C_{free} : \) Final configuration

The task is to compute a path \( \tau : [0, 1] \rightarrow C_{free} \mid \tau(0) = q_I \& \tau(1) = q_G \).

To do this we search for feasible plans and also endure Completeness. This means that the algorithm should return \( \tau \) in “finite time” if it exists or a failure if it does not. This becomes a PSPACE hard problem which is impossible to solve. Luckily, when we look at practical examples in daily life, we seldom encounter situations which do not result in a solution. Thus we can conclude that for average case scenarios, it is possible to solve this problem.

2.1 Discrete search Algorithm

This algorithm aims to solve the Piano Mover’s problem by transforming it into a graph search problem. It converts the world into a space \( X \) which is divided into finite and countable units and then goes through the following procedure:

1. State \( x \) is a state in space \( X \)
2. Transform the world through actions \( u \in U \) (universe of actions).
3. Each action transforms the state $x$ to $x'$. The state transformation equation is: $x' = f(x, u)$

4. Using the initial state $x_I \in X$ and goal state $x_G \in X$ we try to find a set of $u \in U$.

This concept is shown in Figure 7

Figure 6: shows the description of the Piano Movers’ Problem

Figure 7: shows the Piano movers’ problem begin transformed into a graph search problem
2.2 State Transition Graph

To represent the problem a state transition graph is defined using the space $X$ and the universe of actions $U$ such that vertices are represented as $x \in X$ and directed edges exist from $x \rightarrow x' \iff \exists$ an action $u \in U \ | \ f(x, u) = x'$.

Example:

State $x = (i, j)$
$U = (0, 1), \ (0, -1), \ (-1, 0), \ (1, 0)$
($Up \ Down \ Left \ Right$)
State trans : $x' = x + u$
if $x = (3, 4)$ and $u = (0, 1)$
$x' = (3, 4) + (0, 1) = (3, 5)$

Email the TA if you have any questions.