Pareto-Optimal Search over Configuration Space Beliefs for Anytime Motion Planning

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Introduction

For many path planning problems, the execution speedup obtained via the shortest path is often negated by the extra planning effort required to find it. Performing a collision check provides exact information but is computationally expensive. We need to ensure that updating and querying the model is inexpensive. Searching for paths based on collision probability does not guarantee optimality, but may speed up the computation of some feasible path.

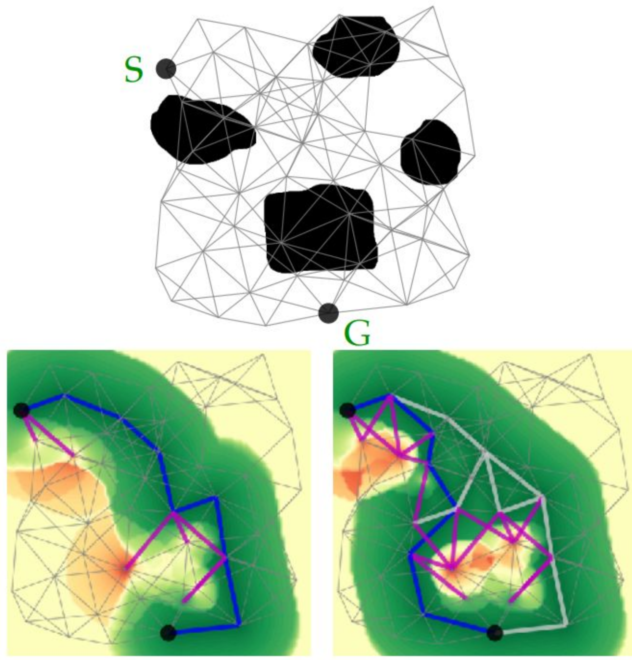
Furthermore, we develop an anytime algorithm to search for successively shorter paths. A number of previous works have addressed this problem - using the probability of collision as a heuristic to guide the search over paths to obtain a feasible path lazily and optimistically searching for the shortest path in a roadmap.

Related Work

- Lazy PRM [Bohlin & Kavraki 2000] Roadmaps with lazy evaluation; search for paths optimistically
- Fuzzy PRM [Nielsen & Kavraki 2000] Assign weights to edges based only on feasibility likelihood Realtime Informed Path Sampling [Knepper & Mason 2011]
- Probabilistically model obstacle locations from collision tests to guide path sampling Instance-based Learning [Pan et al. 2012]
- Use prior collision check results in a k-NN model to reason about unknown configurations

Approach / Method

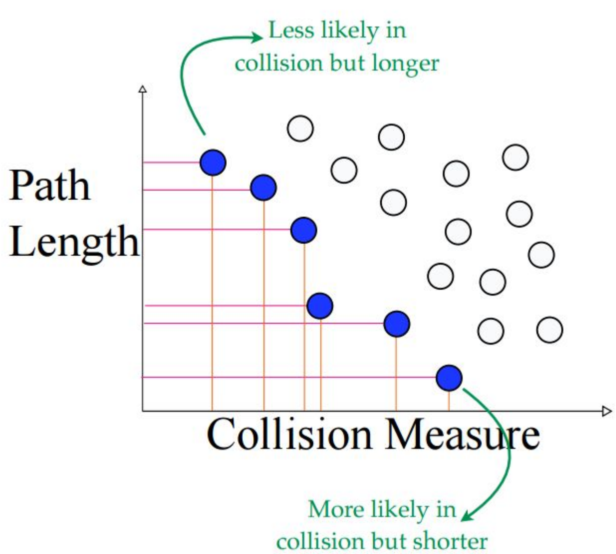
Due to the complexity of the collision-checking problem, probabilistic representations of the world are used to speed up the planning problem. One such model is the **configuration-space (C-Space) Belief Model**, where configurations are mapped to a belief probability.



- This model is much more inexpensive to plan in, but is not certain.
- As the robot travels through configurations, the model is updated with the results of the collision check.

In the paper, there is also the idea of **bi-criteria optimization**, where there are two heuristics which the path is optimizing over. These paths are:

- Short
- Most likely collision free



Below are the two criteria functions for determining these heuristics. This problem needs to deal with pareto optimality, and can be visualized as a **pareto frontier** across the two heuristics.

Length, based on some metric

$$w_l : E \to [0, \infty) \qquad L(\pi) = \sum_{e \in \pi} w_l(e)$$

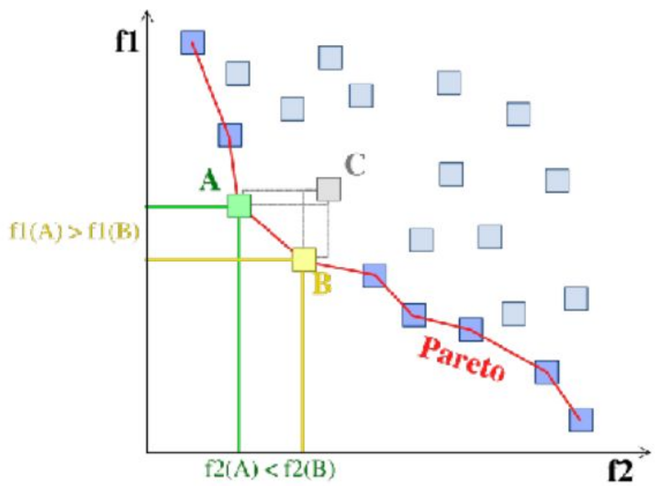
Collision measure

$$w_m : E \to [0, \infty) \qquad M(\pi) = \sum_{e \in \pi} w_m(e)$$

$$w_m(e) = -\log(\rho(e))$$

where $\rho(e)$ is the probability of edge being free

Pareto Optimality: A problem where the state of allocation of resources from which it is impossible to reallocate so as to make any one individual or preference criterion better off without making at least one individual or preference criterion worse off.



In order to trace out this pareto frontier, we need to minimize the objective function:

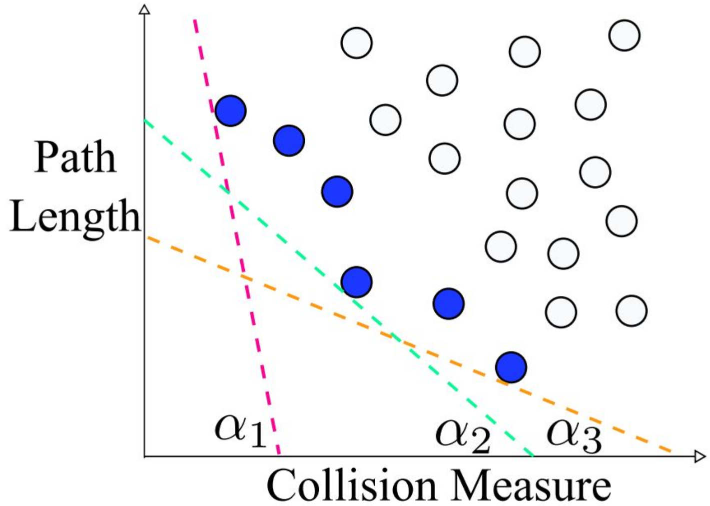
$$\arg\min_{\pi} J^{\alpha}(\pi) = \alpha L(\pi) + (1 - \alpha) M(\pi) \ , \ \alpha \in [0, 1]$$

Additive over edges - dynamic programming for search

Vary α monotonically from 0 to 1 to organize search

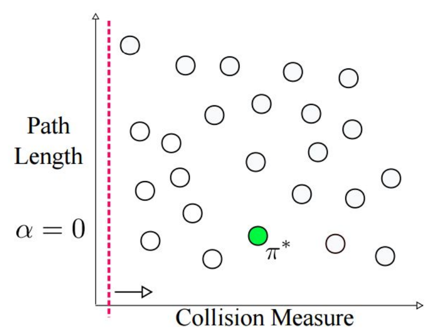
Equivalent to minimizing expected path length under a penalty model

Then we can visually show the convex hull of the **pareto frontier** for different values of alpha.

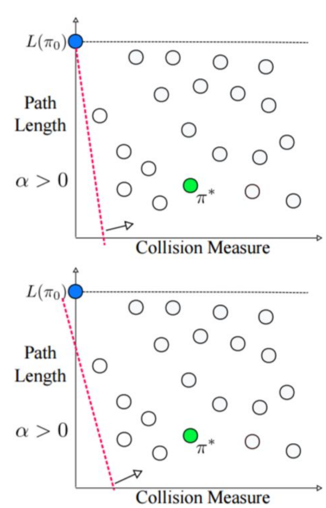


Now we have all we need for the **POMP (pareto-optimal motion planner)**. The algorithm can be visually demonstrated below:

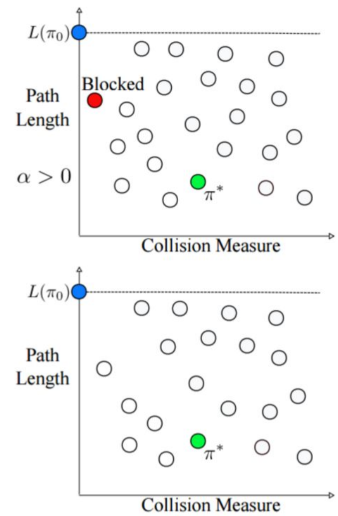
1. Search for path π that minimizes J^{α} ($\alpha = 0$ initially)



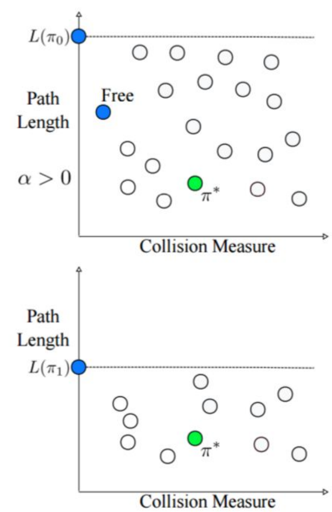
2. If $\pi \equiv \hat{\pi}$ (current best path), increase α and repeat 1



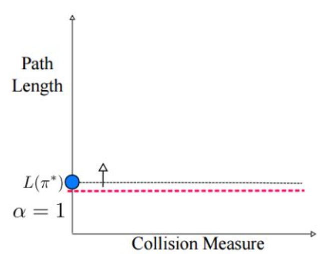
3. Evaluate π for feasibility and update model



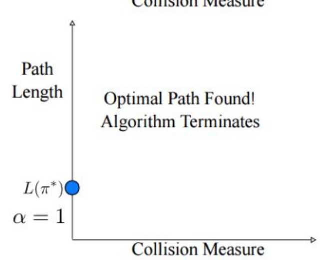
4. If π is feasible, set $\hat{\pi}$ to π, report solution and increase α



5. Repeat 1



Terminate when $\alpha = 1$ and no new candidate paths



Results / Future Work:
The lecture illustrates the challenge for high-DOF robots-No explicit representation of obstacles in 3D world and Expensive collision checking for articulated, complex robots. In order to solve this problem-Obtain successively shorter feasible paths (anytime) while minimizing collision checks, many algorithm are invented, which includes-C-Space Belief Model, Bi-criteria Optimization, Bi-criteria Optimization, Pareto Optimality, Convex Hull of Frontier, POMP and so on. Though many algorithm have been invented, those algorithms are not good enough to solve this problem. Future work will focus on-Incremental sampling when roadmap fails, more complex belief models - manifolds & model reuse for multi-query planning that is robust to small environmental changes

References:

[1] “Pareto-Optimal Search over Configuration Space Beliefs for Anytime Motion Planning”, Shushman Choudhury, Christopher M. Dellin, Siddhartha S. Srinivasa, Carnegie Mellon University <http://www.andrew.cmu.edu/user/shushmac/pomp_iros2016.pdf>

[2] “Pareto-Optimal Search over Configuration Space Beliefs for Anytime Motion Planning”, Shushman Choudhury, Christopher M. Dellin, Siddhartha S. Srinivasa, Carnegie Mellon University <https://personalrobotics.ri.cmu.edu/files/courses/16662/guest_lectures/Shushman_3_8_2017.pdf>