GRASP VERIFICATION

So far we have studied the Had Point Contact Model which operated under the assumption of frictionless contacts. We are now going to generalize it using the Coulomb Friction Model.

Side Notes:

1. Unit Ball <- Pick worst possible scenario
2. Convex Hull <- interior is what matters
3. Some constraints with hard point contact model in general:
   - You can’t exert infinite force using fingers.
   - Fingers can resist some torques better than others which is not taken into account.
   - The possible wrenches that your “opponent” can exert are also limited by the geometry of the object itself.
4. Force Closure v/s Form Closure
   - Force closure implies Form Closure (not necessarily the other way round)

COULOMB FRICTION MODEL

\[ f = \lambda_1 * f_1 + \lambda_2 * f_2, \quad \lambda_1, \lambda_2 \geq 0 \]

\[ \alpha \]

\[ f_1 \quad f_2 \quad f_n \]

\[ f_t \]

\[ \alpha \]

\[ \lambda_1 \quad \lambda_2 \]

Figure 1: Polyhedral Convex cone of friction force.

\( \alpha \) is the angle of repose, \( f_1 \) and \( f_2 \) are the modeled components of the frictional force, \( f_n \) is the normal portion of the force, and \( f_t \) is the tangential portion of the force.
From Figure 1 we have

\[
\frac{f_t}{f_n} = \tan(\alpha) \rightarrow f_t = \tan(\alpha) \times f_n = \mu \times f_n
\]

\(\mu\) is known the coefficient of friction.

Using simple trigonometry, we can plug \(f_1\) and \(f_2\) in to the equation for the force and the wrench:

\[
f_1 = \begin{bmatrix} \sin \alpha \\ \cos \alpha \end{bmatrix}, \quad f_2 = \begin{bmatrix} -\sin \alpha \\ \cos \alpha \end{bmatrix}
\]

\[
f = \begin{bmatrix} \sin \alpha \\ \cos \alpha \end{bmatrix} \times \lambda_1 \ + \ \begin{bmatrix} -\sin \alpha \\ \cos \alpha \end{bmatrix} \times \lambda_2
\]

\[
\omega = \begin{bmatrix} \frac{f_1}{r \times f_1} \times \lambda_1 \ + \ \frac{f_2}{r \times f_2} \times \lambda_2
\]

A matrix \(G\) is defined such that

\[
G \times \lambda = \omega
\]

For each contact point we have two net forces as shown in figure 2.

![Figure 2: Illustration of net forces at each contact point with friction model](image)

In the above drawing, \(G\) is as follows:

\[
G = \begin{bmatrix}
    f_{11} & f_{12} & f_{21} & f_{22} \\
    r_1 \times f_{11} & r_1 \times f_{12} & r_2 \times f_{21} & r_2 \times f_{22}
\end{bmatrix}
\]
Coulomb Friction Model in 3D

In 3D, the friction forces would form a cone in as shown in figure 3 below.

![Figure 3: 3D friction forces form a cone](image)

You can approximate the cone by inscribing inside it a polyhedral convex cone. In other words, convert the ice cream cone to inverted square pyramid.

![Figure 4: Cross Section of a polyhedral convex cone inscribed inside 3D friction cone](image)

We can choose any angle of the form $\frac{2\pi}{n}$ where $n$ is the number of divisions for the inscribed polyhedral convex cone as long as our estimate of $n$ is conservative. The example shown in figure 4 has $n = 4$. As $n \to \infty$ the polyhedral cone converges to the circular cone, which implies that larger $n$ values produce more accurate approximations.
**Nguyen’s Condition:** An object is in *force closure* if each part of the line joining contact points lies inside respective friction cones.

![Figure 5: Figure illustrating Nguyen’s condition for force closure](image)

**GRASP QUALITY METRICS**

Here we ask the question how much better is one grasp than another OR How *good* is a force closure grasp?

The fundamental ambiguity is tat there are different definitions of “better”

![Figure 6: Conversion of forces from contact force space to wrench space using Wrench Map](image)

**Important discussion on matrix multiplication:**

The action of any matrix is to turn a circle in the domain into an ellipse in the range.
The orientation of the ellipsoid so formed depends on the eigen vectors of $A$ (if $A$ is square) and the length of the major and minor axes of the ellipsoid depends on the eigen values of $A$ (if $A$ is square).

**In the case of Grasp Map**

Typically, the Wrench Map ($G$) is non-square, so the length of the major and minor axes depend upon the singular values of the matrix $G$.

The question of Grasp quality effectively boils down to a question of which ellipsoid is better.

**Singular Values of $G$**

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n$$
Possible Grasp Metrics are:

**M1: $\sigma_{min}$**

- Lowest singular value should be as high as possible. It tells you the smallest disturbance wrench you can’t resist

**M2: $\frac{\sigma_{min}}{\sigma_{max}} \in (0,1]$**

- This captures the property of isotropy meaning if the ellipsoid is circular it implies that you are safer in almost all different directions.
- Higher the value of this metric the better it is.

**M3: $Volume(G) = \sqrt{\det(GG^T)}$**

- It captures a general trend. Looks at all singular values in multiple dimensions and not just the maximum and minimum values.

**Notes:**

1. Don’t put full faith in grasp metrics. However, they can be used with machine learning techniques to train the computer what to look for.
2. Another thing that can be tried is weighting different dimensions differently.

**Note:** Until now, we have assumed that our adversary has a unit ball hammer using which he can apply any unit wrench in 3D space. We haven’t taken the geometry or surface properties of the object into account. Using all possible forces that can be exerted on the object, we can get a ‘task wrench space.’ Our force closure condition thus requires our ellipsoid to completely bound the task wrench space.
GRASP PLANNING

While dealing with Grasp Verification, we have lived under the assumption that grasps have been magically generated for us. But, in reality we have to address the question, How do we generate good grasps?

General steps followed in Grasp Planning

In simulation:

1. Pick a pre-grasp configuration
2. Run a controller
3. Verify
4. Loop until success

Pre-grasp configuration: orients hand or how will you attack the object.

Controller: Grasps arms around the object.

Key issues:

- Very slow (modelling physics is computationally expensive)
  - Mitigate with caching all good grasps offline to form a GRASP TABLE: precomputed set of successful pre-grasp configurations for hand-object pair.
  - How do you store these?
  - How do you access them effectively?
  - What happens when the real world is different?
- Key thing: Prioritize grasps so that you quickly figure out which ones will work.
- More to be discussed next week
EXTRA READING

The grasp quality metrics listed above were based on the algebraic properties of the grasp matrix G. Some more metrics can be defined on the basis of the geometric relations of the contact points. To understand that, we need to know what is a grasp polygon. It is a polygon whose vertices are the contact points on the object as can be seen in fig 9.

![Grasp Polygon](image)

**Figure 9: Grasp Polygon**

An index to quantify the uniform distribution of the fingers on the object compares how far are the internal angles of the grasp polygon from those of the corresponding regular polygon. The quality of the grasp under this criterion, called the stability grasp index, is given by

\[
Q = \frac{1}{\theta_{\text{max}}} \sum_{i=1}^{n} |\theta_i - \bar{\theta}|
\]

where \( n \) is the number of fingers, \( \theta_i \) the internal angle at vertex \( i \) of the contact polygon, \( \bar{\theta} \) is the average internal angle of the corresponding regular polygon (given in degrees by \( \bar{\theta} = 180(n - 2)/n \)), and \( \theta_{\text{max}} = (n - 2)(180 - \bar{\theta}) + 2\bar{\theta} \) is the sum of the internal angles when the polygon has the most ill conditioned shape (i.e. degenerates into a line and the internal angles are either 0 or \( \pi \)). The stability index is minimum when the contact polygon is regular; for instance, in a three finger grasp the grasp is optimum when the contact polygon is an equilateral triangle.

Other metrics based on the area of the grasp polygon and the distance between the centroid \( C \) and the center of mass \( CM \) of the polygon can also be defined. For further reading refer to the paper mentioned in the references [1].

References