Robot Autonomy Notes

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Manipulation Planning

There are 2 main components of manipulation planning:

1. **Arm planning:** planning collision-free motions of the arm and hand. This includes collisions between the arm and itself, the arm and the environment, and collision between grasped objects and the environment.
2. **Grasp planning:** finding stable grasps of objects (*this will be the focus of the next few lectures and the first homework)*

Why is arm planning hard? Note that you have to account for "growing an extra limb," i.e. you have to redo kinematics computations whenever you pick something up.

Even though there are discrete planning methodologies for arm and grasp planning, there is an innate tension between them. This makes the two tasks highly interrelated. For instance, a naive grasp planner might come up with good grasps that the arm cannot physically reach.

How do you marry both? Something called manipulation planning.

Grasping

We will start by looking at a pipeline for performing grasping tasks.

1. **Approach:** this involves determining how to bring the gripper within reach of the object
2. **Control:** this involves determining how the hand will close after the approach, intrinsic to world assumptions and grasper

3. **Verification:** this involves ensuring that a grasp is stable. There are a lot of things to consider when determining whether a grasp is stable; e.g., extrinsic agents (gravity), object properties (friction), and antagonistic property (animal running away).

Grasping Philosophies

1. **Planning approach**: This method is programmatic and mathematically backed, so it can be simulated. However, it requires that the designer have very accurate models of the system for good results.
2. **Reactive approach**: This method uses a highly-compliant hand that reacts to the objects it grasps by conforming to the object's geometry. Examples of this style include the SDM Hand[1] and the Universal Gripper[2]. Planning is simplified at the cost of no stability guarantee.
3. **Opportunistic approach**: This method doesn't focus on grasping a particular object, but assumes that a predetermined technique will tend to grasp an object if the environment is sufficiently "target-rich," i.e. there are many objects that can be grasped. If the initial grasp fails, the gripper will try again. The disadvantage of this approach is that it could potentially have a high failure rate and therefore be slow.

It is clear from observing the different approaches that humans use all three, depending on the task. We will analyze grasping using an "adversarial" perspective, whereby we design a system that can withstand any disturbance from an imagined adversary.

Verification

Definition:

- Given a grasp of an object by a hand
  - Object geometry
  - Contact points on the object
- Determine: is the grasp stable?
  - Able to resist arbitrary external force

Contact Model

We will use a frictionless point contact model for our analysis:

[1] https://www.youtube.com/watch?v=RIXxLtcyHAs
[2] https://www.youtube.com/watch?v=0d4f8fEysf8

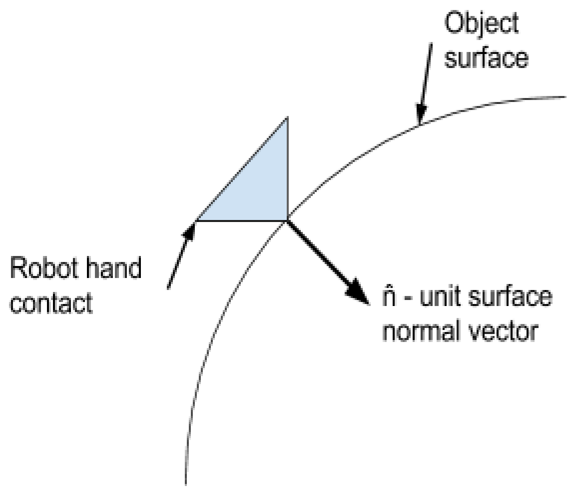

Figure 1: A simple grasp

In this model, the force exerted by the contact point is

$$f = \lambda \hat{n}$$

where

$$\lambda \geq 0$$

This unilateral constraint is a result of the fact that the robot hand can only push, not pull.

1-Dimensional Verification

For a 1-dimensional object, we can devise a grasp with two points of contact:

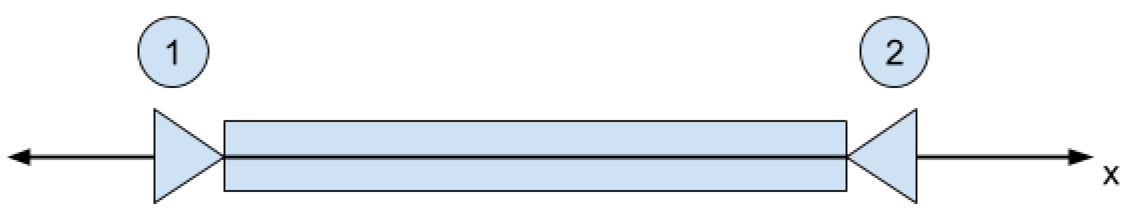

Figure 2: A 1-dimensional grasp

We can then determine the following:

$$\hat{n}_1 = 1 \qquad f_1 = \lambda_1 \qquad \lambda_1 \geq 0$$
$$\hat{n}_2 = -1 \qquad f_2 = -\lambda_2 \qquad \lambda_2 \geq 0$$

and

$$\sum f = f_1 + f_2 = \lambda_1 - \lambda_2$$

Graphing the net force in 1-dimensional force space produces:

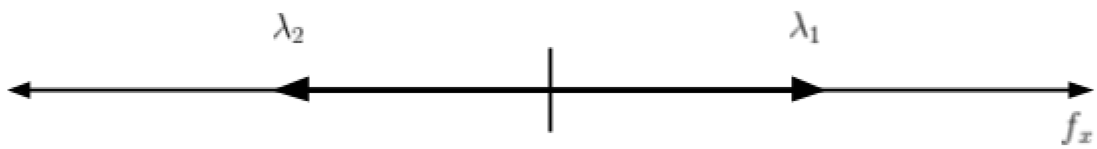

Figure 3: Forces graphed in 1-D force space

Positive Linear Span

- The positive linear span of a set of vectors is the space that can be created by any linear, positively-scaled combination of the vectors
- More formally[3], for the set of vectors $\{\mathbf{v}_i\}$

$$pos(\mathbf{v}_i) = \left\{ \sum k_i \mathbf{v}_i | k_i \geq 0 \right\}$$

We can tell from Figure 3 that the grasp forces span the force space. This means that any external force applied to the object can be counteracted by a combination of the forces exerted by the robot hand, so we can say that this grasp is stable.

[3] http://www.cs.uu.nl/docs/vakken/moma/Lecture13_done.pdf

2-Dimensional Verification

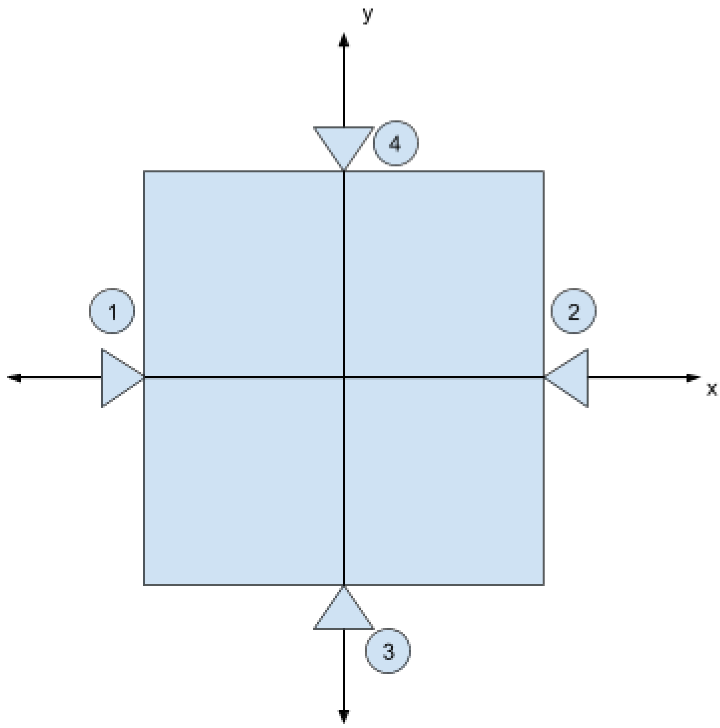

Figure 4: A 2-dimensional problem

In the 2-D problem, the unit normal vectors will have two components. We will represent the unit force in the direction of the normal vector with the letter w. Because $\hat{n}$ is also a unit vector, we can say they are equal:

$$\hat{n} = w$$

The force exerted by the contact points is then given by:

$$\hat{n}_4 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad w_1 = \hat{n}_1 \qquad f_1 = \lambda_1 w_1 \qquad \lambda_1 \geq 0$$

$$\hat{n}_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \qquad w_2 = \hat{n}_2 \qquad f_2 = \lambda_2 w_2 \qquad \lambda_2 \geq 0$$

$$\hat{n}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad w_3 = \hat{n}_3 \qquad f_3 = \lambda_3 w_3 \qquad \lambda_3 \geq 0$$

$$\hat{n}_4 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad w_4 = \hat{n}_4 \qquad f_4 = \lambda_4 w_4 \qquad \lambda_4 \geq 0$$

Summing the forces, we can rewrite them using matrix notation:

$$f = f_1 + f_2 + f_3 + f_4 = \lambda_1 w_1 + \lambda_2 w_2 + \lambda_3 w_3 + \lambda_4 w_4 =$$

$$\begin{bmatrix} w_1 & w_2 & w_3 & w_4 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = G\lambda$$

The matrix of unit forces, G, is called the grasp map. For an external force, f_{ext}, we can generate a grasp map to oppose the force:

$$f_{ext} = -G\lambda \qquad \lambda \geq 0$$

$$\forall f_{ext}$$

This seems to match the definition of stability that we used for one dimension. Can we prove this stability mathematically? For that we have to consider the 3-dimensional problem.

3-Dimensional Verification

By using three dimensions, we can consider the impact of moments. Remember that forces are free vectors, which means that they can be applied anywhere along the line of action and produce the same result on the object.

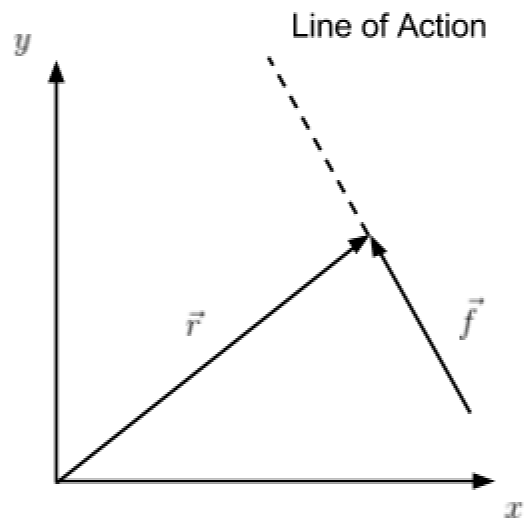

Figure 5: Force $\vec{f}$ creates a moment about the origin

The force in Figure 5 produces a moment about the origin that is the cross product of the moment arm and the force:

$$\vec{m} = \vec{r} \times \vec{f}$$

Figure 6 represents a 2-D grasp that produces nonzero moments. Intuitively this can be seen by the position of contact points 1 and 2. A force on these points produces a moment resulting in a rotation about the z axis counterclockwise.

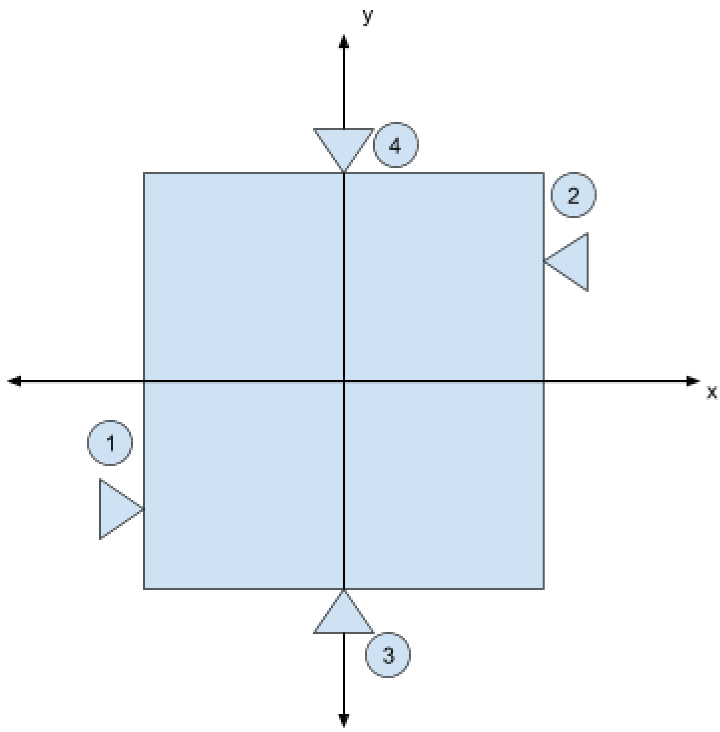

Figure 6: A slightly different grasp in 2-dimensions

The new equations of the Figure 6 configuration are outlined below. We can include the moment in our vector w:

$$w = \begin{bmatrix} \hat{n}_1 \\ \vec{r} \times \hat{n}_1 \end{bmatrix} = \begin{bmatrix} \hat{n}_{1,x} \\ \hat{n}_{1,y} \\ m_1 \end{bmatrix}$$

This new vector that includes the moment is called a wrench.

$$\hat{w}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \hat{r}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \qquad \hat{n}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad f_1 = \lambda_1 w_1 \qquad \lambda_1 \geq 0$$

$$\hat{w}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad \hat{r}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \hat{n}_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \qquad f_2 = \lambda_2 w_2 \qquad \lambda_2 \geq 0$$

$$\hat{w}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \hat{r}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \hat{n}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad f_3 = \lambda_3 w_3 \qquad \lambda_3 \geq 0$$

$$\hat{w}_4 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \qquad \hat{r}_4 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad \hat{n}_4 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad f_4 = \lambda_4 w_4 \qquad \lambda_4 \geq 0$$

We can now graph positive linear span of the forces and moments together in wrench space:

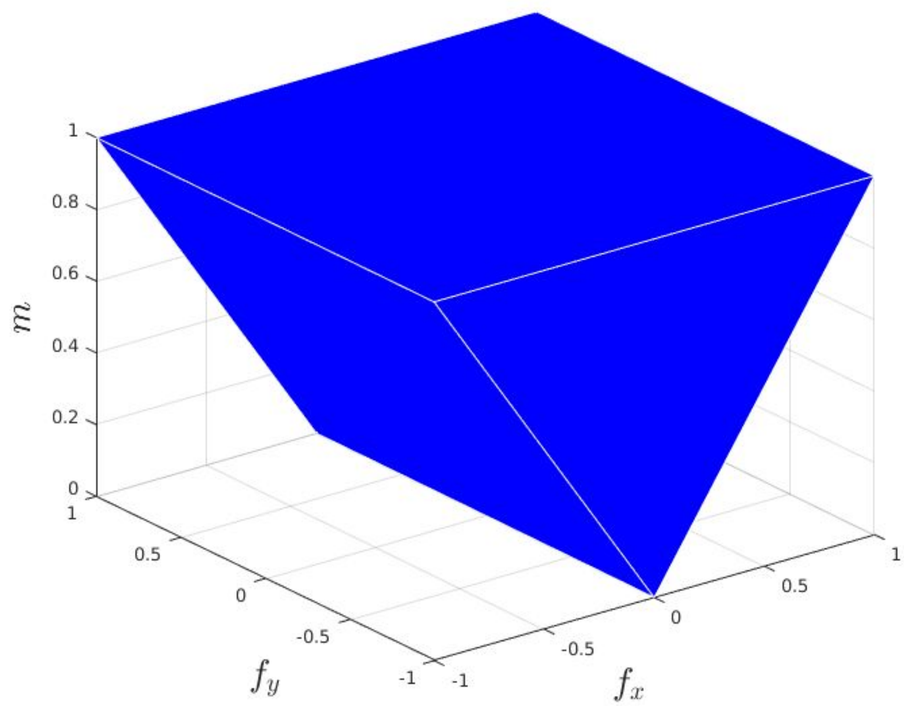

Figure 7: Polyhedral convex cone

The shape formed by the union of the wrenches is called a polyhedral convex cone (PCC). We can see from the graph that, even though the forces exerted by our robot hand span the force-space, they do not span the wrench space, which indicates that the grasp is actually unstable.

Further Considerations

Once the object starts rotating because of the addition of an external force, doesn't our analysis become invalid?

Imagine a robot hand pressing down on a penny that is resting on its edge. Now imagine the same situation, but the penny is replaced with a billiard ball. From a force closure perspective, both situations are equally stable. But it is apparent to a human observer that they aren't. How do we compensate for the difference?

Force closure analysis is instantaneous, so it has limitations. You may need to consider form closure analysis to fully understand the stability of the situation.