30 January 2017
Clare Cui (ccui)
Logan Wan (lwan)
Yoga Nadaraajan (yyn)

16-662 Robot Autonomy: Grasp Verification

Given some grasp, how do we evaluate its stability? There are two basic approaches: force closure, and form closure.

Force Closure

Given a grasp and:

1. The geometry of an object
2. The contact points

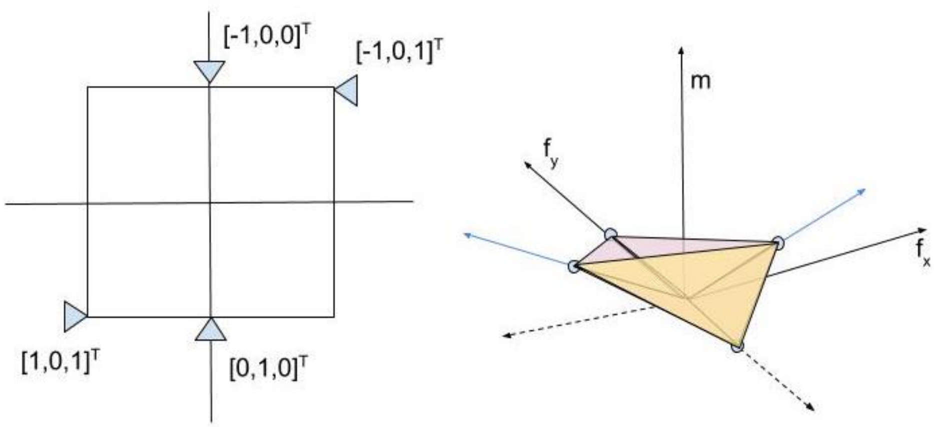

Figure 1: A grasp with its wrench space diagram

Is the grasp stable? In order to find out, we need to evaluate:

$\forall W_{ext}, \exists \lambda \geq 0$, such that $G\lambda = -W_{ext}$

(For all external forces, there exists some *lambda* ≥ 0 such that the grasp map G times *lambda* is equal to the external forces)

More explicitly, given a set of contact points fixed on a rigid body, a grasp is in force closure if an external wrench needs to be applied to keep the body in equilibrium; this, however, does not need to take care of infinitesimally small motions of the object.[http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=163781]

However, is there an easier way to evaluate and ensure that our grasp is stable? How do we compare grasps?

Intuitively, you can hold anything by holding onto it really tightly, which would generate an internal force. This means that you need to create an internal set of forces and moments that span $\mathbb{R}^3$. There are two ways to check this:

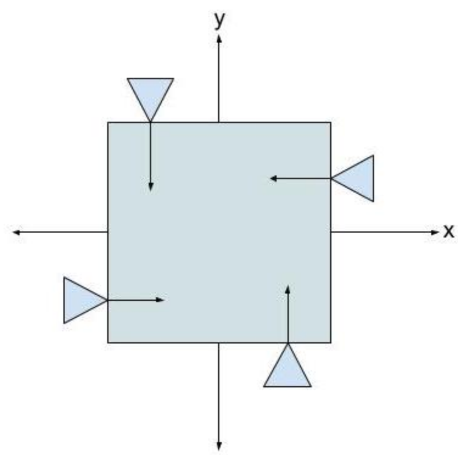

Figure 2: A grasp not in force closure

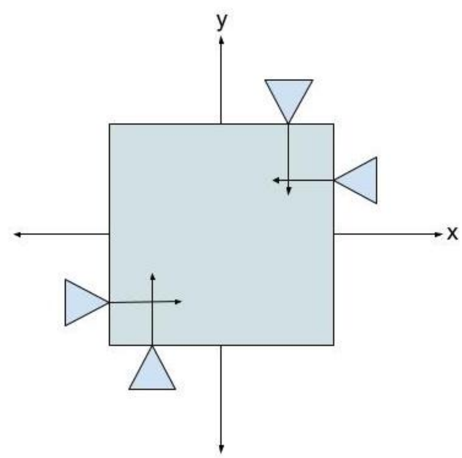

Figure 3: A grasp in force closure

1. **Origin lies in the interior of the convex hull of unit wrenches.**
   See figure 4.

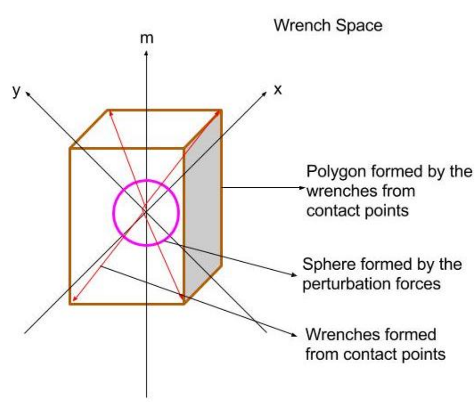

Figure 4: Origin within the convex hull of wrenches

In this example, the origin lies in the interior of the convex hull. Therefore, it does satisfy the next criteria.

2. $rank(G) = 3$ **(in 2D)**

   $rank(G) = 6$ **(in 3D)**

   In 2D, the wrench is:

$$\begin{bmatrix} F_x \\ F_y \\ m \end{bmatrix}$$

   In 3D, the wrench is:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ m_x \\ m_y \\ m_z \end{bmatrix}$$

Form Closure

In form closure, the joints of a manipulator are locked around an object with its palm fixed in space [gra()], meaning the object is totally constrained and is unable to be manipulated, even in an infinitesimally small way. Relating back to force closure, it is possible for both form closure and force closure to exist within the same grasp. However, form closure does not imply force closure.

As an example of form closure, you can visualize an object with a shape that completely encloses it. If the object is able to move within the shape, then it is not in form closure. As the enclosing shape moves closer to the box and eventually eliminates the space in between, form closure is achieved. (Figure 5)

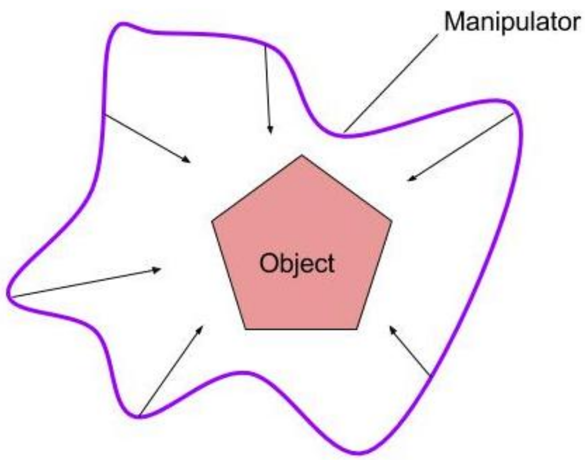

Figure 5: Visual example of form closure

A minimum of 4 points are needed for form closure in 2D, and a minimum of 7 wrenches for form closure in 3D.

Any matrix turns a sphere into an ellipsoid, and this is true for any grasp map G in wrench space. If we have two grasp maps in force closure, we can visualize them as follows in Figure 6:

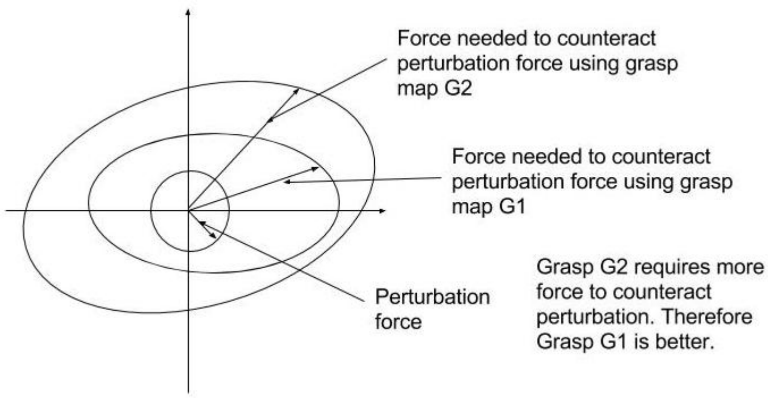

Figure 6: Two grasps G1 and G2 that exert different amounts of force to correct for the same perturbation forces

We can compare these two and see that G_1 requires much less force than G_2 while still maintaining form closure. To prove this mathematically, we can perform a singular value decomposition (SVD) of G to obtain three vectors:

$$SVD(G) = U, \Sigma, V$$

Where:

$$\Sigma = [\sigma_1, \sigma_2, \sigma_3 \geq 0]$$

The minimum value of σ (typically the last one in numpy, MATLAB) corresponds to the smallest wrench value of the grasp.

Thus there are two possible metrics for evaluating these types of grasps:

1. Metric 1: The value of σ_{min}, which corresponds to a grasp of minimum effort.
2. Metric 2: The volume of the ellipsoid:
$$V = \sqrt{det(GG^T)}$$

References

[gra()] Grasping. `https://www.cs.rpi.edu/twiki/pub/RoboticsWeb/LabPublications/PTspringer08.pdf`.