

# Robot Autonomy Lecture Notes

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## Abstract

## Motion Planning

### 1 Configuration

An entity or mathematical construct which unifies all robots. A complete specification of the position of every point in the system.

### 2 Incomplete Taxonomy of Motion Planning

#### 2.1 Plans

3 Types exist: -

- Discrete: Set of discrete actions and resultants which are fixed. E.g. Rubik's Cube, STRIPS.
- Continuous: Needs continuity in trajectory for movement. E.g. Piano movers' problem.
- Hybrid: A mix of discrete and continuous motion and resultant configurations. E.g. Building a robot to solve a chess board. While playing the game itself has discrete motions, the joint angles leading to the robot manipulating the chess pieces is continuous in nature.

#### 2.2 Environment

We need to ensure that the robot does not collide with the surrounding objects. The environment can be broadly classified into the following categories -

- Immovable objects - Static objects such as walls which can not be moved
- Moving objects - People or objects which change their state in the environment over time
- Movable objects - Objects which are static, but can be manipulated by robot-object interaction.

The whole paradigm of motion planning depends upon the fact that we have movable objects so that we can perform manipulation task. It is also key to note that performing manipulation of object at one location can affect our motion planning at a different location. A good example would be: Lifting and moving a table in a room can affect our motion planning at the door.

#### 2.3 Uncertainty

Accounting for uncertainty can be crucial in making our algorithms robust. For explanation purpose, we divide uncertainty of location of an object(cup) in 4 categories-

- No uncertainty - We are very sure about the location of the target object(X)(need figure here)

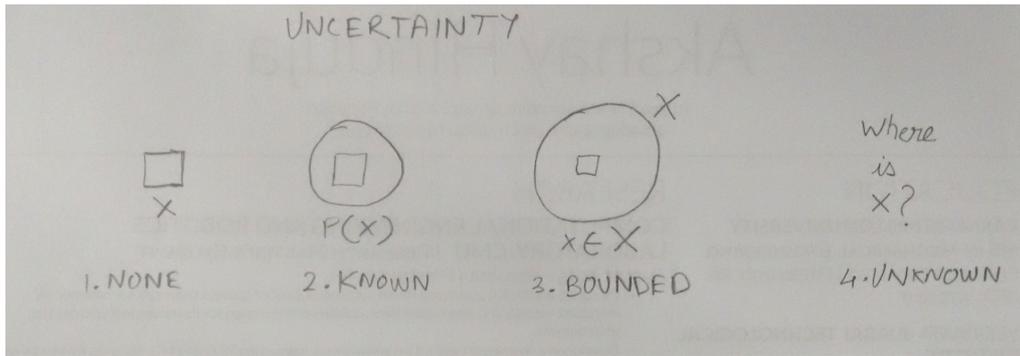


Figure 1: Visual Representation of Uncertainty.

- Known uncertainty - We know that the random variable(X) follows a known probability distribution  $p(x)$ (E.g Gaussian distribution) (need figure here)
- Bounded uncertainty - We are told that the object resides in a bounded space.  $x \in X$
- Unknown uncertainty - We have no idea about the location of the target object. Not even its distribution

Most technical papers categorized based on the 4 taxonomies mentioned above. However, there are restrictions on adhering to these 4 paradigms. Some papers define their own taxonomy  
Eg: Perception

### 3 Configuration Space

Configuration space is a way of representing a robot in an abstract space such that we can achieve generality of adding multiple such robots to the same space. In other words, it's just a way of providing an API for robot interface. We define  $C$  as our configuration space such that  $q \in C$

What is  $q$ ? It's basically all the data required to describe the robot. Formally, it's the minimum data needed to be transmitted for 2 robots to be in the same configuration space. We define a rendering function  $A$  which renders our robot from configuration space to world space  $R^3$  or  $R^2$

$$A(q) : C \rightarrow W$$

Example: A point in the 2D configuration space is defined as  $q = (x, y) \in R^2$  which maps to a disk of radius  $r$  in the configuration space through a rendering function  $A$ .

Example: A two-link arm in configuration space is defined as  $(\theta_1, \theta_2) \in S^1 \times S^1$  which can be mapped to the world space as shown in the figure below

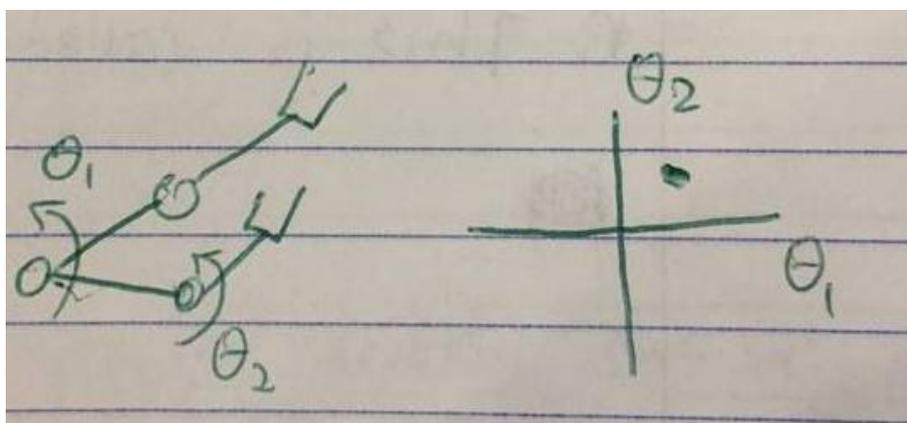


Figure 2: Two-DOF arm in configuration space.

The catch: Apply transformation to all the objects in the configuration space(including the robot and the obstacles)

Let us now define the obstacle space  $O \subset W$

$$C_{obs} = \{q \in C \mid A(q) \cap O \neq \emptyset\}$$

Thus the free configuration space can be denoted as  $C_{free} = C - C_{obs}$

### 3.1 The Minkowski Difference

Intuitively, minkowski difference states that the mapping of the robot from world space to configuration space shrinks the robot while the obstacle being mapped to configuration space dilates. This can be clearly explained through the example below: Example: Consider a disk of radius  $r$  representing a robot and a square shaped obstacle in the world space. The closet position the robot can go is marked around the square such that the disk just touches the robot. When we map this obstacle to the configuration space using the inverse of the rendering function, the obstacle dilates *MoreaboutMinkowski - [https://en.wikipedia.org/wiki/Minkowski\\_addition](https://en.wikipedia.org/wiki/Minkowski_addition)* (figure)

Formally we define the Minkowski difference as -

$$\begin{aligned} C_{obs} &= O \ominus A(o). \\ &= \{(a - o) \mid o \in O \text{ and } \alpha \in A(o)\} \end{aligned}$$

### 3.2 Classwork Problems

- 1D world example

Consider a point in 1D space, which can be represented as a disc of radius  $r$  in 2D space.

$$q = (x, y) \in \mathbb{R}^2$$

Its configuration in 2D is  $(x,y)$ , and rendering in a 2D world is a disc or radius  $r$ .

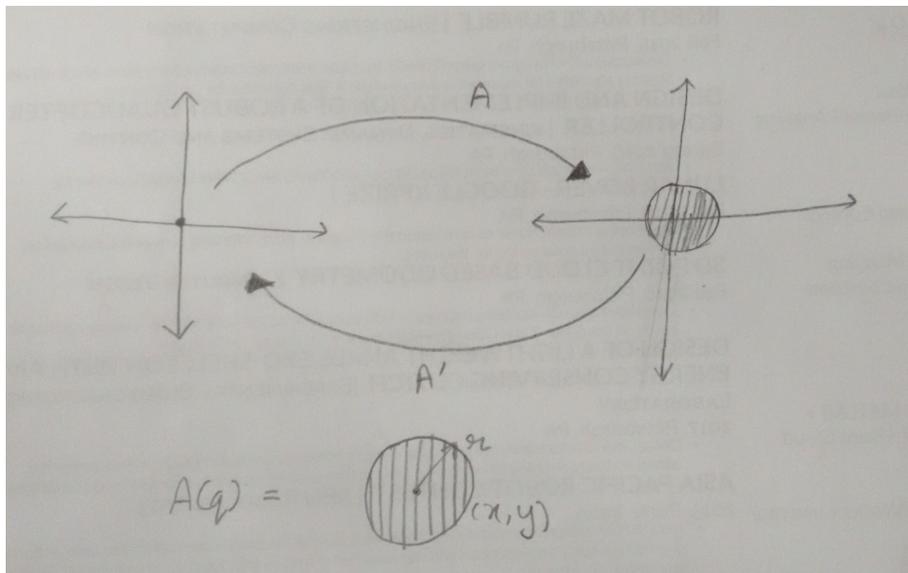


Figure 3: Example 1.

- 2D world example

### 3.3 Topology

## Topology Example

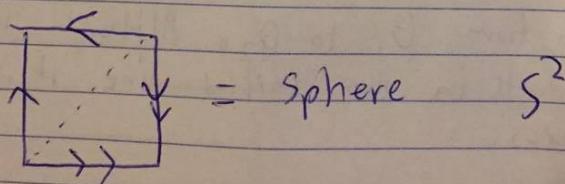
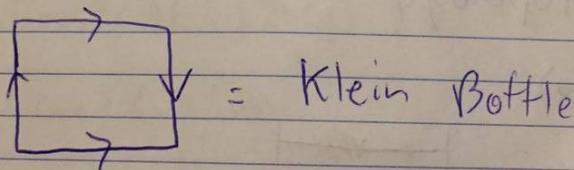
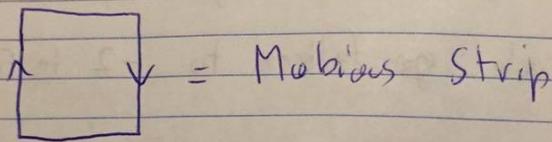
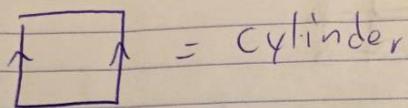


Figure 4: Various Topology Examples