16-843 – Manipulation Algorithms
Grasping

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Summary from last Lecture

**Grasp Synthesis**
Find suitable set of contacts for given object.

**Analytic**
Mathematical modeling of grasp in terms of contact points (and their forces) to evaluate if grasp completely restrains grasp.

**Data Driven**
Look up or predict suitable grasps based on object features.
Summary from last Lecture

**Definition Grasp**: Set of contacts on the object surface that constrain the potential movements of the object in the event of external disturbances.

**Grasp synthesis**: Given an object and some constraints, find a suitable set of contacts.

**Grasp analysis**: Given the object and contact points, decide if the grasp is stable.
Contacts between finger and objects are idealized point contacts
Ignore kinematics of finger
Object is fixed
Only consider transmission of forces between contact points and object
Summary from last Lecture

Coordinate frames for contact and object forces

Contact coordinate frame: $z$-axis points in the direction of the inward surface normal

Object reference: center of mass

Force applied by contact is modeled as a wrench $F_{Ci}$ applied at origin of $C_i$

$$F_{Ci} = \begin{pmatrix} f_i \\ \tau_i \end{pmatrix}$$
**Contact Model**: Maps forces transmitted through the contact to wrench $w_i$ relative to object

- **Frictionless point contact**
- **Point contact with friction**
- **Soft-finger contact**

Wrench applied to object in $C_i$

\[
F_{C_i} = B_{C_i} f_{C_i}
\]

- $f_{C_i} \in FC_{C_i}$
- Friction cone
- Wrench basis
Contact wrench acting on object

\[ F_{O_i} = \text{Ad}^T_{g_{O_i}^{-1}} F_{C_i} = \begin{bmatrix} R_{O_i} & 0 \\ \hat{p}_{O_i} R_{O_i} & R_{O_i} \end{bmatrix} B_{C_i} f_{C_i}, \]

Maps the wrench in the contact frame \( C_i \) to the object frame!

Wrench from contact point \( i \) acting on the object.

Cross product of level arm and transformed force (=torque)

\[ p = [p_1, p_2, p_3]^T \]

\[ \hat{p} = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix} \]

How does it look in 2D?

\[ F_{O_i} = G_i f_{C_i} \]

\[ G_i := \text{Ad}^T_{g_{O_i}^{-1}} B_{C_i} \]
Contact wrench acting on object

\[ F_{O_i} = \text{Ad}_{g_{oc_i}}^T F_{C_i} = \begin{bmatrix} R_{OC_i} & 0 \\ \hat{p}_{OC_i} R_{OC_i} & R_{OC_i} \end{bmatrix} B_{C_i} f_{C_i}, \]

The net object wrench for \( k \) finger contacts is

\[ F_O = G_1 f_{C_1} + G_2 f_{C_2} + \ldots + G_k f_{C_k} = \begin{bmatrix} G_1 & \ldots & G_k \end{bmatrix} \begin{bmatrix} f_{C_1} \\ \vdots \\ f_{C_k} \end{bmatrix} = G f_C \]

Grasp map

\[ f_C \in FC \subset \mathbb{R}^m \]

\[ p = 3 \quad \text{in 2D world} \]
\[ p = 6 \quad \text{in 3D world} \]
Grasp Map

Whiteboard Example

Grasp map for **friction-less point contacts**

\[
G_i = \begin{bmatrix}
R_{Ci} & 0 \\
\hat{p}_{Ci}R_{Ci} & 1
\end{bmatrix} \quad G = \begin{bmatrix}
n_{C_1} \\
p_{C_1} \times n_{C_1} \\
\vdots \\
p_{C_k} \times n_{C_k}
\end{bmatrix}
\]

\[
p = 3 \quad (2D \text{ world})
\]

\[
F_{C_i} = G_i f_{C_i}
\]

\[
G_i := Ad^T_{goc} B_{C_i}
\]

\[
B = \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]

The same for 3D World
Grasp Map

Whiteboard Example

Grasp map for friction-less point contacts
\( p=3 \quad (2D \text{ world}) \)

1. Position of frame origins \( C_i \) in \( O \)
\( p_1=[-1,-a] \quad p_2=[1,a] \)
\( p_3=[0,1] \quad p_4=[0,-1] \)

2. Wrench basis in \( C \)
\[
B = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T
\]

We apply only force in \( y \) in the contact frame

3. Wrench in \( C_i \)
\[
F_{C_1} = B f_{C_1} \quad F_{C_2} = B f_{C_2} \quad F_{C_3} = B f_{C_3} \quad F_{C_4} = B f_{C_4}
\]
Grasp Map

Whiteboard Example

Grasp map for friction-less point contacts

\( p = 3 \) (2D world)

4. Compute object wrench

\[
F_{O_i} = \begin{bmatrix}
R_{OC_i} & 0 \\
\hat{p}_{OC_i} R_{OC_i} & R_{OC_i}
\end{bmatrix}
B_{C_i} f_{C_i}
\]

\( \hat{p}_1 = [a, -1] \)

\[
A_1 = \begin{bmatrix}
0 & 1 & 0 \\
-1 & 0 & 0 \\
1 & a & 1
\end{bmatrix}
\]

\( \hat{p}_2 = [-a, 1] \)

\[
A_2 = \begin{bmatrix}
0 & -1 & 0 \\
1 & 0 & 0 \\
1 & a & 1
\end{bmatrix}
\]
Grasp Map

Whiteboard Example

Grasp map for friction-less point contacts
\[ p = 3 \quad (2D \text{ world}) \]

4. Compute object wrench

\[
F_{Oi} = \begin{bmatrix}
R_{OC_i} & 0 \\
\hat{p}_{OC_i} R_{OC_i} & R_{OC_i}
\end{bmatrix} B_{Ci} f_{Ci}
\]

\[
\hat{p}_3 = [-1, 0] \quad A_3 = \begin{bmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
1 & 0 & 1
\end{bmatrix}
\]

\[
\hat{p}_4 = [1, 0] \quad A_4 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
1 & 0 & 1
\end{bmatrix}
\]
Grasp Map

Whiteboard Example

Grasp map for friction-less point contacts
\[ p = 3 \quad \text{(2D world)} \]

4. Compute object wrench

\[
F_{O_i} = \begin{bmatrix} F_{O_1} \\ F_{O_2} \\ F_{O_3} \\ F_{O_4} \end{bmatrix} = \begin{bmatrix} R_{OC_i} & 0 & A_i \\ \hat{p}_{OC_i} R_{OC_i} & R_{OC_i} & B_{C_i} f_{C_i} \end{bmatrix}
\]

\[
F_{O_1} = \begin{bmatrix} 1 \\ 0 \\ a \end{bmatrix} f_{C_1}, \quad F_{O_2} = \begin{bmatrix} -1 \\ 0 \\ a \end{bmatrix} f_{C_2}, \quad F_{O_3} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} f_{C_3}, \quad F_{O_4} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} f_{C_4}
\]
Grasp Map

Whiteboard Example

Grasp map for friction-less point contacts
\( p=3 \) (2D world)

5. Grasp map

\[
G = \begin{bmatrix}
1 & -1 & 0 & 0 \\
0 & 0 & -1 & 1 \\
a & a & 0 & 0
\end{bmatrix}
\]
Grasp Map

Whiteboard Example

Soft Finger Grasp

\( p = 6 \) (3D world)

\[
B = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
G = \begin{bmatrix}
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
-a & 0 & 0 & 0 & 0 & a & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
0 & a & 0 & 0 & -a & 0 & 0 & 0 \\
\end{bmatrix}
\]

\( p_1 = [0, -a, 0] \)
\( p_2 = [0, a, 0] \)

[Murray, 1994]
Analytical grasp synthesis

Grasp Analysis:
Given the object and contact points, decide if grasp is stable

Form closure:
Constrain the movement of the object

Force Closure
∀\(F_e \in \mathbb{R}^6\) \(\exists f_c : Gf_c = -F_e\)
\(f_c \in FC\)
Force Closure

Whiteboard Example
Grasp map for friction-less point contacts $p=3$ (2D world)

Is this a force closure grasp?

$$\forall F_e \in \mathbb{R}^3 \ \exists f_c : \ Gf_c = -F_e$$

Grasp Map

$$G = \begin{bmatrix}
1 & -1 & 0 & 0 \\
0 & 0 & -1 & 1 \\
\alpha & \alpha & 0 & 0
\end{bmatrix}$$

No:

$f_{ci}$ needs to be positive.

Not able to resist positive torque:

$$F_e = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}$$
A grasp is force-closure iff

1) \( \text{rank}(G) = p \)

2) There exist a vector of contact forces \( f_N \in N(G) \) s.t. \( f_N \in \text{int}(FC) \)

\( G f_N = 0 \)

Interior of the friction cone

Internal forces: contact forces which result in no net force on the object
Convexity conditions for force-closure grasps for frictionless point contacts [Murray, 1994]

Let $G$ in $\mathbb{R}^{p \times m}$ be the associated grasp matrix and let $\{G_i\}$ denote the columns of $G$. The following statements are equivalent:

(1) The grasp is force closure
(2) The columns of $G$ positively span $\mathbb{R}^p$
(3) The convex hull of $\{G_i\}$ contains a neighborhood of the origin.
Force Closure

Interactive 3D pen demonstration

\[
G = \begin{bmatrix}
1 & -1 & 0 & 0 \\
0 & 0 & -1 & 1 \\
a & a & 0 & 0
\end{bmatrix}
\]
Force Closure

Whiteboard Example + Interactive 3D pen demonstration

\[ G = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ a & a & -b & -b \end{bmatrix} \]
Force Closure

Projected into 2D (arbitrary G with 5 contacts)
Force Closure

Using convexity for grasps with friction

Forces in the friction cone can be written as positive linear combinations of edges of cone

\[
G = \begin{bmatrix}
1 & 1 & -1 & -1 \\
\mu & -\mu & -\mu & \mu \\
\mu r & -\mu r & -\mu r & \mu r \\
F_{11} & F_{12} & F_{21} & F_{22}
\end{bmatrix}
\]
Force Closure

Using convexity for grasps with friction

In 3D the friction cone cannot be represented as sum of finite set of vectors

Solution: Approximate cone with finite set of vectors

[Murray, 1994]
How can we check for force closure?

Algorithm:
1) **Approximate the friction cone at each contact with a set of wrenches**
2) Combine all wrenches into a set of points $S$ in wrench space
3) Compute convex hull of points in $S$
4) Verify that origin is in convex hull
Force Closure

How to rank the grasp? Which grasp is better?

Need grasp quality metric!
Grasp Quality metrics

A) [Ferrari and Canny] Radius of largest hyper-sphere you can fit in convex hull centered in origin

B) [Zhu and Wang]: Numerical test which measures the scaling factor for the maximum compact set inscribed in the grasp wrench space.
Active Force Closure

[Shapiro, 2001]

Defines as active force closure, the condition we have discussed as force closure!

Why active?
Passive Force Closure

If we can balance the external wrenches without changing actively the forces applied at the contact points.
Grasp Synthesis

Find all stable [force closure] grasp for a given object.

Naïve: For a hand with n fingers, search for set of n point contact on the object surface.

Disadvantages:
• Takes a long time
• Ignores hand kinematic
Grasp Synthesis

Approximations

[Miller et al., 2003]

1. Approximate shape of object with simple shape primitives
2. Sample pose of hand relative to object with finger in pre-shape
3. Approach object until contact and close the fingers
4. Test contact points for force closure and evaluate (rank)

Disadvantages?
GRASPING WITH HUMANS IN THE LOOP
Grasping Is Challenging

- Grasp algorithms must be able to:
  - ensure force closure
  - avoid obstacles
  - allow for future object use

Miller et al., 2003

not a good grasp for drinking
Challenge: Grasps do not happen in isolation; they are part of a larger series of complex manipulations.
Humans in the Loop (HitL)

• People are very good at:
  – perceiving and classifying objects
  – planning multi-step tasks
  – selecting grasp locations

• Human in the loop grasping combines people’s finely honed cognitive abilities with robot autonomy

Applications for HitL Grasping

- When would you want to add a human into the loop for grasping?

High-risk activities

Robonaut 2 (Diftler et al., 2011)

Extending human capabilities

Kinova JACO (courtesy of Kinova, Inc.)
Interfaces for HitL Grasping

• How much human control vs. robot autonomy?

Direct Control

- User directly teleoperates robot through input interface

Point and click interface (Leeper at al., 2012)

Joystick interface (Herlant et al., 2016)
Interfaces for HitL Grasping

• How much human control vs. robot autonomy?

Waypoint Following

- User sets *waypoints* and the robot autonomously interpolates between them (Leeper et al., 2012)
Waypoint Following

At the DARPA Robotics Challenge Trials in 2013:
• Waypoint following was the most autonomous control method used
• Teams with greater levels of robot autonomy tended to do better

https://www.youtube.com/watch?v=2nFwbyfQAkM

Interfaces for HitL Grasping

• How much human control vs. robot autonomy?

Grasp Execution

- Operator sets only final grasp pose, and the robot autonomously plans to that pose while avoiding collisions

Interfaces for HitL Grasping

- How much human control vs. robot autonomy?

Grasp Planning

- Robot autonomously suggests grasp poses, user accepts one, robot autonomously plans to that pose

(Leeper et al., 2012)
Interfaces for HitL Grasping

• How much human control vs. robot autonomy?
  – strategies with more robot autonomy enabled more grasps
  – strategies with more robot autonomy resulted in fewer collisions

Direct Control
Waypoint Following
Grasp Execution
Grasp Planning

Waypoint A -> Waypoint B

Control Spectrum

All figs (Leeper et al., 2012)
Other Human-Robot Grasping Problems

- **Joint manipulation** – autonomous robot and human physically sharing loads to jointly manipulate an object (e.g., Lawitzky, Mörtl, Hirche (2010), “Load Sharing in Human-Robot Cooperative Manipulation,” *RO-MAN.*)
Other Human-Robot Grasping Problems

• **Handovers** – autonomous robot and human transferring objects from one to another
  
  (e.g., Edsinger & Kemp (2007), “*Human-Robot Interaction for Cooperative Manipulation: Handing Objects to One Another,*” *RO-MAN.*)